**Gauss-Jordan Matrix Elimination**

- This method can be used to solve systems of linear equations involving two or more variables. However, the system must be changed to an augmented matrix.

- This method can also be used to find the inverse of a 2x2 matrix or larger matrices, 3x3, 4x4 etc.

  **Note:** The matrix must be a square matrix in order to find its inverse.

An **Augmented Matrix** is used to solve a system of linear equations.

- [ ] $a_1x + b_1y + c_1z = d_1$
- $a_2x + b_2y + c_2z = d_2$
- $a_3x + b_3y + c_3z = d_3$

- When given a system of equations, to write in augmented matrix form, the coefficients of each variable must be taken and put in a matrix.

For example, for the following system:

- $3x + 2y - z = 3$
- $x - y + 2z = 4$
- $2x + 3y - z = 3$

  **Augmented Matrix**

  $$
  \begin{bmatrix}
  3 & 2 & -1 & 3 \\
  1 & -1 & 2 & 4 \\
  2 & 3 & -1 & 3 \\
  \end{bmatrix}
  $$
-There are three different operations known as **Elementary Row Operations** used when solving or reducing a matrix, using Gauss-Jordan elimination method.

1. Interchanging two rows.
2. Add one row to another row, or multiply one row first and then adding it to another.
3. Multiplying a row by any constant greater than zero.

**Identity Matrix** is the final result obtained when a matrix is reduced. This matrix consists of ones in the diagonal starting with the first number.

- The numbers in the last column are the answers to the system of equations.

\[
\begin{bmatrix}
1 & 0 & 0 & 3 \\
0 & 1 & 0 & 2 \\
0 & 0 & 1 & 5 \\
\end{bmatrix}
\text{— Identity Matrix for a 3x3}
\]

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 2 \\
0 & 1 & 0 & 0 & 6 \\
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 4 \\
\end{bmatrix}
\text{— Identity Matrix for a 4x4}
\]

- The pattern continues for bigger matrices.

**Solving a system using Gauss-Jordan**

The best way to go is to get the ones first in their respective column, and then using that one to get the zeros in that column.

- It is very important to understand that there is no exact procedure to follow when using the Gauss-Jordan method to solve for a system.

\[
\begin{align*}
3x + 2y - z &= 3 \\
x - y + 2z &= 4 \\
2x + 3y - z &= 3 \\
\end{align*}
\]

*Write as an augmented matrix.*

\[
\begin{bmatrix}
3 & 2 & -1 & 3 \\
1 & -1 & 2 & 4 \\
2 & 3 & -1 & 3 \\
\end{bmatrix}
\text{— Switch row 1 with row 2 to get a 1 in the first column}
\]
\[
\begin{bmatrix}
1 & -1 & 2 & 4 \\
3 & 2 & -1 & 3 \\
2 & 3 & -1 & 3
\end{bmatrix}
\]

Multiply row 1 by -3 and add to row 2 to get a zero

Row 1 multiplied by -3 \rightarrow -3 3 -6 -12
Row 2 \rightarrow + 3 2 -1 3
New Row 2 \rightarrow 0 5 -7 -9

-Put the new row 2 in the matrix, note that though row 1 was multiplied by -3, row 1 didn’t change in our matrix.

\[
\begin{bmatrix}
1 & -1 & 2 & 4 \\
0 & 5 & -7 & -9 \\
2 & 3 & -1 & 3
\end{bmatrix}
\]

Using a similar procedure of multiplying and adding rows, obtain the following matrix

\[
\begin{bmatrix}
1 & -1 & 2 & 4 \\
0 & 5 & -7 & -9 \\
2 & 3 & -1 & 3
\end{bmatrix}
\]

Multiply row 1 by -2 and add to row3 as above.

\[
\begin{bmatrix}
1 & -1 & 2 & 4 \\
0 & 5 & -7 & -9 \\
0 & 5 & -5 & -5
\end{bmatrix}
\]

Switch rows 2 and 3 to obtain the following

\[
\begin{bmatrix}
1 & -1 & 2 & 4 \\
0 & 5 & -7 & -9 \\
0 & 5 & -5 & -5
\end{bmatrix}
\]

Divide the second row by 5 to obtain a 1 in the second row.

\[
\begin{bmatrix}
1 & -1 & 2 & 4 \\
0 & 1 & -1 & -1 \\
0 & 5 & -7 & -9
\end{bmatrix}
\]

Add row 2 to row 1

\[
\begin{bmatrix}
1 & 0 & 1 & 3 \\
0 & 1 & -1 & -1 \\
0 & 5 & -7 & -9
\end{bmatrix}
\]

Multiply and add like we did earlier, -5*R2+R3
\[
\begin{bmatrix}
1 & 0 & 1 & \| & 3 \\
0 & 1 & -1 & \| & -1 \\
0 & 0 & -2 & \| & -4 \\
\end{bmatrix}
\]

\textit{Divide row 3 by -2 to obtain a 1 in the third row.}

\[
\begin{bmatrix}
1 & 0 & 1 & \| & 3 \\
0 & 1 & -1 & \| & -1 \\
0 & 0 & 1 & \| & 2 \\
\end{bmatrix}
\]

- Finally, the matrix can be solved in two different ways:

A. Using the 1 in column 3, obtain the other zeros and the solutions.

\[
\begin{bmatrix}
1 & 0 & 0 & \| & 1 \\
0 & 1 & 0 & \| & 1 \\
0 & 0 & 1 & \| & 2 \\
\end{bmatrix}
\]

\[x = 1 \quad y = 1 \quad z = 2\]

B. Solve by using back substitution.

- The solution to the last row is \( z = 2 \), the answer can be substituted into the equation produced by the second row. \( y - z = -1 \). Substituting into this equation, it simplifies to:

\[y - 2 = -1\]
\[y = 1\]

- Again, substituting the answer for \( z \) into the first equation will give the answer for \( x \).

\[x + z = 3\]
\[x + 2 = 3\]
\[x = 1\]