Function Gallery: Some Basic Functions and Their Properties

**Linear Equation** \( y = mx + b \)
This Example: \( y = 3x + 3 \)
Domain \((-\infty, \infty)\)
Range \((-\infty, \infty)\)
No Symmetry

**Linear Equation** \( y = -mx + b \)
This Example: \( y = -x + 0 \)
Domain \((-\infty, \infty)\)
Range \((-\infty, \infty)\)
Symmetric about the origin

**Linear Equation: Horizontal Line**
This Example: \( y = 4 \)
Domain \((-\infty, \infty)\)
Range \(\{4\}\)
Symmetric about the y-axis

**Linear Equation: Vertical Line**
This Example: \( x = 4 \)
Domain \(\{4\}\)
Range \((-\infty, \infty)\)
Symmetric about the x-axis

**Absolute Value Function:** \( y = |x| \)
Domain \((-\infty, \infty)\)
Range \([0, \infty)\)
Increasing on \((0, \infty)\)
Decreasing on \((-\infty, 0)\)

**Absolute Value Equation:** \( x = |y| \)
Domain \([0, \infty)\)
Range \((-\infty, \infty)\)
Increasing on
Decreasing on

**Absolute Value Equation** \( y = -|x| \)
This Example: \( y = -|x| + 2 \)

**Absolute Value Equation** \( x = -|y| \)
**Quadratic Function** \( y = x^2 \)

- **Domain** \((-\infty, \infty)\)
- **Range** \([0, \infty)\)
- **Increasing on** \([0, \infty)\)
- **Decreasing on** \((-\infty, 0]\)
- **Symmetric about the y-axis**

![Quadratic Function Graph](image1)

**Quadratic Function** \( y = -x^2 \)

- **Domain** \((-\infty, \infty)\)
- **Range** \((-\infty, 0]\)
- **Increasing on** \((-\infty, 0)\)
- **Decreasing on** \([0, \infty)\)
- **Symmetric about the x-axis**

![Quadratic Function Graph](image2)

**Radical Function** \( y = \sqrt{x^2} \)

- **This Example:** \( y = \sqrt{16 - x^2} \)

![Radical Function Graph](image3)

**Radical Function** \( y = -\sqrt{x^2} \)

- **This Example:** \( y = -\sqrt{4 - x^2} \)

![Radical Function Graph](image4)

**Rational Functions** \( y = \frac{1}{x} \)

- **This Example:** \( y = \frac{2x + 5}{x - 1} \)

![Rational Functions Graph](image5)

**Exponential Function** \( y = e^x \)

- **This Example:** \( y = 2^x \)

![Exponential Function Graph](image6)
**Square Root Function** \( y = \sqrt{x} \)

This Example: \( y = \sqrt{x} \)
Domain \([0, \infty)\)
Range \([0, \infty)\)

**Square Root Function** \( y = -\sqrt{x} \)

This Example: \( y = -\sqrt{3x - 2} \)

**Cubic Function** \( y = x^3 \)

Domain \((-\infty, \infty)\)
Range \((-\infty, \infty)\)
Increasing on \((-\infty, \infty)\)
Symmetric about the origin

**Cubic Function** \( y = -x^3 \)

Domain \((-\infty, \infty)\)
Range \((-\infty, \infty)\)
Decreasing on \((-\infty, \infty)\)
Symmetric about the origin

**Cube Root Function** \( y = \sqrt[3]{x} \)

This Example: \( y = \sqrt[3]{x} - 5 \)
Note: a cube root function is the inverse of a cubic function

**Quartic or Fourth-Degree Function** \( y = x^4 \)

This Example: \( y = x^4 - x^3 - 7x^2 + x + 6 \)

**Quartic or Fourth Degree Function** \( y = -x^4 \)

This Example: \( y = -x^4 - 4x^3 - 6x^2 - 4x - 1 \)
Parabolas
Translations & Transformations

\[ y = ax^2 + bx + c \quad \text{or} \quad y = a(x - h)^2 + k \]

\[ y = x^2 \quad y = -x^2 \quad x = y^2 \quad x = -y^2 \]
Logarithmic Functions

Basic Logarithmic Graph \( y = \ln(x) \)

Below are some different examples of some basic logarithmic functions and their graphs.
Exponential Functions

Basic Exponential Graph  \( y = e^x \)

- Range: \([0, \infty)\)
- Domain: \((-\infty, \infty)\)

Below are some different examples of some basic exponential functions and their graphs.
Rational Functions $y = \frac{1}{x}$

The graph of a rational function is called a hyperbola

Rational Function: $y = \frac{2x + 5}{x - 1}$

Rational Function: $y = \frac{-2x - 2}{x - 1}$

Rational Function: $y = \frac{x^2 + 3}{x - 1}$

Rational Function: $y = \frac{x^3 - 8}{x^2 + 5x + 6}$

Rational Function: $y = \frac{x^2 + 2x - 3}{x^2 - 5x - 6}$

Rational Function: $y = \frac{x^2 + x - 2}{x^2 - x - 2}$
Trigonometric Functions

The Table below outlines each change for each trigonometric ratio.

<table>
<thead>
<tr>
<th>FUNCTION</th>
<th>PERIOD</th>
<th>AMPLITUDE</th>
<th>HORIZ. SHIFT*</th>
<th>VERT. SHIFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = a \sin (bx + c) + d )</td>
<td>( 2\pi )</td>
<td>(</td>
<td>a</td>
<td>)</td>
</tr>
<tr>
<td>OR ( y = a \cos (bx + c) + d )</td>
<td>(</td>
<td>b</td>
<td>)</td>
<td></td>
</tr>
<tr>
<td>( y = a \tan (bx + c) + d )</td>
<td>( \frac{\pi}{</td>
<td>b</td>
<td>} )</td>
<td>---</td>
</tr>
<tr>
<td>OR ( y = a \cot (bx + c) + d )</td>
<td>(</td>
<td>b</td>
<td>)</td>
<td></td>
</tr>
<tr>
<td>( y = a \sec (bx + c) + d )</td>
<td>( 2\pi )</td>
<td>---</td>
<td>( \frac{a}{b} )</td>
<td>( d )</td>
</tr>
<tr>
<td>OR ( y = a \csc (bx + c) + d )</td>
<td>(</td>
<td>b</td>
<td>)</td>
<td></td>
</tr>
</tbody>
</table>
Asymptotes
Use the steps below to find asymptotes:

**Asymptotes:**
Factor and reduce the rational function first. If a factor is eliminated in that reduction, it determines a hole in the graph.

**Vertical Asymptotes**
To find the Vertical Asymptotes and Domain set the denominator equal to zero and solve.

Example: Find the domain and vertical asymptotes

\[ y = \frac{x + 2}{x^2 + 2x - 8} \]

Domain: \( x \neq -4, 2 \)  
Vertical Asymptotes: \( x = -4, 2 \)

**Horizontal Asymptotes**
To Find the Horizontal Asymptotes, compare the degree of the leading coefficients in the numerator and the denominator.

- If the degrees of the numerator and denominator are the same, \( p(x) = q(x) \)
  Then, the horizontal asymptote is the ratio of the leading coefficients.

  Example:\[ \frac{2x + 1}{x + 3} \]
  Because the degree of \( 2x \) and \( x \) is the same, then the H.A. is found by finding the ratio of the leading coefficients, which in this example is \( \frac{2}{1} \) which equals 2.

  So the **Horizontal Asymptote is** \( y = 2 \)

- If the numerator's degree is less than the denominator, \( p(x) < q(x) \)
  then the x-axis is the horizontal asymptote and the equation is \( y = 0 \)

  Example:\[ \frac{x}{x^2 - 4} \]
  because the degree of \( x \) is less than the degree of \( x^2 \) then the **HA is** \( y = 0 \)

- If the numerator is greater than the denominator, \( p(x) > q(x) \) then there's no Horizontal Asymptote.

**Slant/Oblique Asymptotes**
To Find the Slant/Oblique Asymptote compare the degree of the leading coefficients in the numerator and denominator.

If the **numerator's degree is greater (by a margin of 1)**
you have a slant asymptote which you will find by doing long division.

Example:\[ \frac{2x^2 + 3x - 5}{x + 2} \]
because the degree of \( 2x^2 \) is greater than the degree of \( x \) then you have a slant asymptote. You must use long division or synthetic division to find the slant asymptote.

Step 1: \( x + 2 \sqrt{2x^2 + 3x - 5} \)

Step 2: The answer is \( 2x - 1 + \frac{-3}{x + 2} \)

Step 3: The Horizontal Asymptote is \( y = 2x - 1 \)

**Note:** you can not have a slant asymptote and a horizontal asymptote together.
Function Shifts and Transformations

If you've been doing your graphing by hand, you've probably started noticing some relationships between the equations and the graphs. The topic of function transformation makes these relationships more explicit.

Let's start with the function notation for the basic quadratic: \( f(x) = x^2 \). A function transformation takes whatever basic function \( f(x) \) and then "transforms" it, which is a fancy way of saying that you change the formula a bit and thereby move the graph around.

For instance, the graph for \( x^2 + 3 \) looks like this

![Graph of \( x^2 + 3 \)](image)

This is three units higher than the basic quadratic. That is, \( x^2 + 3 = f(x) + 3 \). We added a "3" outside the basic squaring function \( f(x) = x^2 \) to go from the basic quadratic \( x^2 \) to the transformed function \( x^2 + 3 \).

This is always true: To move a function up, you add outside the function. That is, \( f(x) + b \) is \( f(x) \) moved up \( b \) units. Moving the function down works the same way; \( f(x) - b \) is \( f(x) \) moved down \( b \) units.

On the other hand, \((x + 3)^2\) looks like this:

![Graph of \((x + 3)^2\)](image)

In this graph, \( f(x) \) has been moved over three units to the left. That is, \( f(x + 3) = (x + 3)^2 \) is \( f(x) \) shifted three units to the left.

This is always true: To shift a function left, add inside the function's argument. That is, \( f(x + b) \) gives \( f(x) \) shifted \( b \) units to the left. Shifting to the right works the same way; \( f(x - b) \) is \( f(x) \) shifted \( b \) units to the right.