Exponents

Common Mistakes
Exponents – Zero and First Power

How to use the Zero and First Power

- **Zero Exponent Rule**
  \[ x^0 = 1 \]
  
  *Examples*
  
  \[ 4^0 = 1 \]
  
  \[ (-3)^0 = 1 \]

- **First Power**
  \[ x^1 = x \]
  
  *Examples*
  
  \[ 5^1 = 5 \]
  
  \[ (-2)^1 = -2 \]

Common Mistakes

- **Zero Exponent Rule**
  
  - Writing the answer as the original problem.
    - Incorrect: \[ 4^0 = 4 \]
    - Correct: \[ 4^0 = 1 \]

- **First Power**
  
  - Writing 1 as the answer.
    - Incorrect: \[ 4^1 = 1 \]
    - Correct: \[ 4^1 = 4 \]
Exponents – Product Rule

How to use the Product & Quotient Rule

- **Product Rule**
  
  \[ x^n \cdot x^m = x^{n+m} \]
  
  **Examples**
  
  \[ x^3 \cdot x^5 = x^{3+5} = x^8 \]
  
  \[ 3^2 \cdot 3^1 = 3^{2+1} = 3^3 = 27 \]

- **Quotient Rule**
  
  \[ \frac{x^n}{x^m} = x^{n-m} \]
  
  **Examples**
  
  \[ \frac{x^5}{x^3} = x^{5-3} = x^2 \]

  \[ \frac{4^3}{4^2} = 4^{3-2} = 4^1 = 4 \]

Common Mistakes

- **Product Rule**
  
  - **Multiplying the exponents**
  
    **Incorrect:** \( x^3 \cdot x^6 = x^{3+6} = x^{18} \)

    **Correct:** \( x^3 \cdot x^6 = x^{3+6} = x^9 \)

- **Quotient Rule**
  
  - **Dividing the exponents**
  
    **Incorrect:** \( \frac{4x^6}{2x^2} = 2x^3 \)

    **Correct:** \( \frac{4x^6}{2x^2} = 2x^4 \)
Exponents – Power Rule

How to use the power rule

- **Power Rule - Product**
  \[(a^m b^n)^n = a^{mn} b^{mn}\]
  - Ex.
  \[(2x^3)^3 = 2^{1\cdot3} \cdot x^{3\cdot3} = 8x^9\]

- **Power Rule - Quotient**
  \[\left(\frac{a^m}{b^n}\right)^n = \frac{a^{mn}}{b^{mn}}\]
  - \[\left(\frac{3^2}{x^5}\right)^3 = \frac{3^{2\cdot3}}{x^{5\cdot3}} = \frac{3^6}{x^{15}} = \frac{729}{x^{15}}\]

Common Mistakes

- **Applying the power rule incorrectly to coefficients** (the numbers in front of the variables).
  - Hint: You never multiply an exponent by a coefficient. Exponents can only add, subtract, multiply, and divide by other exponents.
    - **Incorrect:** \[(2x^3)^3 = 2 \cdot 3 \cdot x^{3\cdot3} = 6x^9\]
    - **Correct:** \[(2x^3)^3 = 2^{1\cdot3} \cdot x^{3\cdot3} = 8x^9\]

- **Adding the exponents instead of multiplying them.**
  - **Incorrect:** \[(2x^3)^3 = 2^{1+3} \cdot x^{3+3} = 8^4 \cdot x^6 = 16x^6\]
  - **Correct:** \[(2x^3)^3 = 2 \cdot 3 \cdot x^{3\cdot3} = 6x^9\]
Exponents – Negative Exponents

How to use the negative exponent rule

- **Negative Exponent Rule(s)**

  \[ a^{-m} = \frac{1}{a^m} \]

  **Hint:** Move the number/variable to the opposite place to remove the exponent.
  - If the no. with the negative exponent is in the top, move it to the bottom to get rid of the negative on the exponent.
  - If the no. with the negative exponent is in the bottom, move it to the top to get rid of the negative on the exponent.

  \[ \left( \frac{a}{b} \right)^{-m} = \left( \frac{b}{a} \right)^m \]

  **Hint:** If there is a fraction that has a negative exponent, flip the entire fraction. This removes the negative sign from the exponent.

- **Common Mistakes**

  - Applying the negative exponent rule to everything in the problem, instead of just the variable that has the negative exponent.
    - **Incorrect:** \( 2x^{-3} = \frac{1}{2x^3} \)
      - The 2 has an exponent of 1, not -3. The negative exponent rule does not apply to the 2.
    - **Correct:** \( 2x^{-3} = \frac{2}{x^3} \)

  - When flipping the fraction, getting rid of all the negative exponents, not just the negative exponent on the outside of the parenthesis.
    - **Incorrect:** \( \left( \frac{2y^{-4}}{x} \right)^{-3} = \left( \frac{x}{2y^4} \right)^3 = \frac{x^3}{2^3 y^{12}} = \frac{x^3}{8y^{12}} \)
      - Moving the “y” to the bottom when you are fixing the negative on the 3 does not remove the negative. It must be fixed in a different operation
    - **Correct:** \( \left( \frac{2y^{-4}}{x} \right)^{-3} = \left( \frac{x}{2y^{-4}} \right)^3 = \frac{x^3}{2^3 y^{-12}} = \frac{x^3 y^{12}}{8} \)
Exponents - Combination

How to work with multiple Exponent Rules

- You can apply any exponent rule in any order as long as it does not break the Order of Operation.

  Ex. \( \left( \frac{3x^6}{x^4y^8} \right)^2 \)

  In this problem you need to use the Power Rule, and the Quotient Rule. It does not matter which you use first.

\[
\left( \frac{3x^6}{x^4y^8} \right)^2 = \left( \frac{3x^2}{y^8} \right)^2 = \frac{3^2x^{22}}{y^{82}} = \frac{9x^4}{y^{16}}
\]

- Ex. \( 2x^2(3x^3)^2 \)

  In this example, you must apply the power rule before you use the product rule, otherwise you break the Order of Operation that states, powers before multiplication.

\[
2x^2(3x^3)^2 = 2x^2 \cdot 9x^6 = 18x^{2+6} = 18x^8
\]

Common Mistakes

- Not memorizing the rules.
  - Hint: here are some sayings to help
    - When you multiply you add exponents.
    - When you divide you subtract exponents
    - When you use the power rule you multiply exponents
    - When you have a negative exponent, fix it first.

- Applying the rules so that it breaks the Order of Operation, PEMDAS.

- Getting confused as to which rule needs to be applied.

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