

Basic Rules for Algebra & Logarithmic Functions

Quadratic Formula:	Example:
If $p(x) = ax^2 + bx + c$, $a \neq 0$ and $0 \leq b^2 - 4ac$, then the real zeroes of p are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.	If $p(x) = x^2 + 3x - 1 = 0$, with $a=1$, $b=3$, and $c=-1$, then $p(x) = 0$ if $x = \frac{-3 \pm \sqrt{13}}{2}$
Special Factors:	Examples:
$x^2 - a^2 = (x - a)(x + a)$	$x^2 - 9 = (x - 3)(x + 3)$
$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$	$x^3 - 8 = (x - 2)(x^2 + 2x + 4)$
$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$	$x^3 + 27 = (x + 3)(x^2 - 3x + 9)$
Binomial Theorem	Examples:
$(x + a)^2 = x^2 + 2ax + a^2$	$(x + 3)^2 = x^2 + 2(x)(3) + 3^2 = x^2 + 6x + 9$
$(x - a)^2 = x^2 - 2ax + a^2$	$(x - 5)^2 = x^2 - 2(x)(5) + 5^2 = x^2 - 10x + 25$
$(x + a)^3 = x^3 + 3ax^2 + 3a^2x + a^3$	$(x + 2)^3 = x^3 + 3(x)^2(2) + 3(x)(2)^2 + 2^3 = x^3 + 6x^2 + 12x + 8$
$(x - a)^3 = x^3 - 3ax^2 + 3a^2x - a^3$	$(x - 1)^3 = x^3 - 3(x)^2(1) + 3(x)(1)^2 - 1^3 = x^3 - 3x^2 + 3x - 1$
Factoring by Grouping	Example:
$acx^3 + adx^2 + bcx + bd = ax^2(cx + d) + b(cx + d) = (ax^2 + b)(cx + d)$	$3x^3 - 2x^2 - 6x + 4 = x^2(3x - 2) - 2(3x - 2) = (x^2 - 2)(3x - 2)$

Arithmetic Operations:			
$ab + ac = a(b + c)$	$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$	$\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$	$\left(\frac{\frac{a}{b}}{\frac{c}{d}}\right) = \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$
$a\left(\frac{b}{c}\right) = \frac{a}{1} \times \frac{b}{c} = \frac{ab}{c}$	$\frac{a - b}{c - d} = \left(\frac{-1}{-1}\right) \frac{a - b}{c - d} = \frac{b - a}{d - c}$	$\frac{ab + ac}{a} = \frac{a(b + c)}{a} = b + c, a \neq 0$	$\left(\frac{\frac{a}{b}}{c}\right) = \frac{a}{b} \div \frac{c}{1} = \frac{a}{b} \times \frac{1}{c} = \frac{a}{bc}$
$\frac{a}{\left(\frac{b}{c}\right)} = \frac{a}{1} \div \frac{b}{c} = \frac{a}{1} \times \frac{c}{b} = \frac{ac}{b}$			

Exponents and Radicals			
$a^0 = 1, a \neq 0$	$\frac{a^x}{a^y} = a^{x-y}$	$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$	$\sqrt[n]{a^m} = a^{m/n} = \left(\sqrt[n]{a}\right)^m$
$a^{-x} = \frac{1}{a^x}$	$(a^x)^y = a^{xy}$	$\sqrt{a} = a^{1/2}$	$\sqrt[n]{ab} = \sqrt[n]{a} * \sqrt[n]{b}$
$a^x a^y = a^{x+y}$	$(ab)^x = a^x b^x$	$\sqrt[n]{a} = a^{1/n}$	$\sqrt[n]{\left(\frac{a}{b}\right)} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Algebraic Errors to Avoid:	
$\frac{a}{x+b} \neq \frac{a}{x} + \frac{a}{b}$	To see this error, let $a, b,$ and, x all equal to 1. Then, $\frac{1}{2} \neq 2.$
$\sqrt{x^2 + a^2} \neq x + a$	To see this error, let $x = 3$ and $a = 4.$ Then, $5 \neq 7.$
$a - b(x-1) \neq a - bx - b$	Remember to distribute the negative signs. The equation should be $a - b(x-1) = a - bx + b$
$\left(\frac{x}{a}\right) \frac{bx}{b} \neq \frac{bx}{a}$	To divide fractions, invert and multiply. The equation should be $\frac{x}{a} \frac{bx}{b} = \frac{x}{a} \left(\frac{1}{b}\right) = \frac{x}{ab}$
$\sqrt{-x^2 + a^2} \neq -\sqrt{x^2 - a^2}$	We can't factor a negative sign outside of the square root.
$\frac{a+bx}{a} \neq 1+bx$	This is one of many examples of incorrect cancellation. By applying the properties above, the equation should be $\frac{a+bx}{a} = \frac{a}{a} + \frac{bx}{a} = 1 + \frac{bx}{a}.$
$(x^2)^3 \neq x^5$	In this case we multiply the exponents, then the correct equation is $(x^2)^3 = x^2 x^2 x^2 = x^{2+2+2} = x^6$

LOGARITHMIC RULES

Logarithmic function	$y = \log_a x,$ defined by $x = a^y$
Common logarithm	$\log x = \log_{10} x$
Natural logarithm	$\ln x = \log_e x$
Log Properties	
	$\log_a a^x = x; a^{\log_a x} = x$
	$\log_a (MN) = \log_a M + \log_a N$
	$\log_a (M / N) = \log_a M - \log_a N$
	$\log_a (M)^N = N(\log_a M)$
Change of base formulas	
	$\log_b x = \frac{\log x}{\log b}$
	$\log_b x = \frac{\ln x}{\ln b}$