Ratios, Rates, and Proportions

COMMON MISTAKES
Ratios

How to Translate Ratios

- A ratio is a quotient of two numbers or the quotient of two quantities that have the same units.
- Three ways to write a ratio:
  The ratio of $a$ to $b$
  
  $a$ to $b$ or $a:b$ or $\frac{a}{b}$

Common Mistakes

- Writing the ratio with reversed values.
- Writing the ratios without the same units.
- Incorrect:
  
  The ratio of 5 to 9 $\Rightarrow$ 9 : 5 or $\frac{9}{5}$
  
  The ratio of 12 ounces to 2 pounds $\Rightarrow$ 12 : 2 or $\frac{12}{2}$

- Correct:
  
  The ratio of 5 to 9 $\Rightarrow$ 5 : 9 or $\frac{5}{9}$
  
  The ratio of 12 ounces to 2 pounds $\Rightarrow$
  
  $\frac{12 \text{ ounces}}{2 \text{ pounds}} = \frac{12 \text{ ounces}}{2(16 \text{ ounces})} = \frac{3}{8}$ or 3 : 8
How to Translate Rates

- A rate is a quotient of two quantities that have *different units*.

- **Unit rate** is the rate for a single unit.
  
  $1.38 for 6 apples = $1.38/6 apples = $0.23/1 apple = 23 cents per apple

Common Mistakes

- Not reducing a unit rate to a single unit in the denominator.

  Incorrect:

  Unit rate for 30 hamburger patties to 20 buns:
  
  \[
  \frac{30 \text{ patties}}{20 \text{ buns}} = \frac{3 \text{ patties}}{2 \text{ buns}}
  \]

  Mistake: Not reduced to unit rate.

  Correct:

  Unit rate for 30 hamburger patties to 20 buns:
  
  \[
  \frac{30 \text{ patties}}{20 \text{ buns}} = \frac{3 \text{ patties}}{2 \text{ buns}} = 1.5 \text{ patties per 1 bun} 
  \]
Proportions

- A *proportion* is a mathematical statement that two ratios or two rates are equal.
- Each of the four numbers of a proportion are terms: First \((a)\) and fourth terms \((d)\) are *extremes*; Second \((b)\) and third terms \((c)\) are means.
- In a proportion, the product of the extremes is equal to the *product of the means*.

\[
\text{If } ad = bc, \text{ then } \frac{a}{b} = \frac{c}{d}.
\]

**Common Mistakes**

- Setting up proportion incorrectly.
- Not checking to see if solution results in a true proportion.
- Incorrect:
  
  If 6 apples cost $1.38, how much does it cost for 16 apples?
  
  \[
  \frac{6 \text{ apples}}{16 \text{ apples}} = \frac{c}{\$1.38}
  \]
  
  Solve: \(\frac{3}{2} = \frac{9}{x}\)
  
  You found \(x\) to be 18 \(\Rightarrow\) Check: \(3(18) \neq 2(9)\)

- Correct:
  
  If 6 apples cost $1.38, how much does it cost for 16 apples?
  
  \[
  \frac{6 \text{ apples}}{16 \text{ apples}} = \frac{16 \text{ apples}}{\$1.38}
  \]
  
  Solve: \(\frac{3}{2} = \frac{9}{x}\)
  
  You found \(x\) to be 6 \(\Rightarrow\) Check: \(3(6) = 2(9)\)