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**EXPONENTS**

**Exponents** are used to write long multiplications in a short way. The exponent will tell you how many times the number or variable needs to be multiplied. In this case the variable is ‘a’.

**Examples:**

\[(6)(6) = 6^2 \quad \text{and} \quad a \cdot a \cdot a \cdot a = a^4\]

In exponential notation, the number or variable being multiplied several times is called the **base**. The exponent, or number that tells you how many times you need to multiply, is called the **power**.

\[2^3 \rightarrow \text{2 is the base, and 3 is the power}\]

Exponents are mostly used when dealing with variables, or letters, since it is easier and simpler to write \[x^4\] than \[x \cdot x \cdot x \cdot x\]. Also, letters are used for they represent an unknown number, and so, the term: variable.

There are a few rules used for simplifying exponents:

**Zero Exponent Rule** – Any number or letter raised to the zero power is always equal to 1.

**Example:**

\[3^0 = 1 \quad \text{and} \quad a^0 = 1\]

**Product Rule** – When multiplying the same base, the exponents are added together.

**Example:**

\[x^3 \cdot x^4 \quad \leftarrow \text{Same base, x, add the exponents, } 3 + 4 = 7\]

\[x^3 \cdot x^4 = x^7\]

**Quotient Rule** – When dividing the same base, subtract the exponents.

**Example:**

\[\frac{x^5}{x^2} \quad \leftarrow \text{Same base, x, subtract the exponents, } 5 - 2 = 3\]

\[\frac{x^5}{x^2} = x^3\]
**Power Rule** – When the operation contains parentheses, multiply the exponent on the inside with the exponent on the outside.

*Example:* \((y^6)^2 \leftarrow \text{Multiply the exponents, } 6 \times 2 = 12\)

\((y^6)^2 = y^{12}\)

When there is a fraction inside the parentheses, the exponent multiplies on the current power of the numerator and the denominator. However, this rule does not apply if you have a sum or difference within the parentheses; in that case a different rule will apply.

*Examples:*

\[
\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}
\]

\[
\left(\frac{x^3}{y^2}\right)^4 = \left(\frac{x^3}{y^2}\right)^4 = \frac{x^{12}}{y^8}
\]

! Be careful: \((\frac{a + b}{c + d})^2\) is not the same as \(\frac{a^2 + b^2}{c^2 + d^2}\)!

In fact:

\[
\left(\frac{a + b}{c + d}\right)^2 = \frac{a^2 + 2ab + b^2}{c^2 + 2cd + d^2}
\]

**Negative Signs** – If the negative sign is outside the parentheses, perform the operations inside the parentheses and carry out the negative sign to the final answer.

*Example:* 

\(-3^3 = -(3)(3)(3) = -(27) = -27\)

However, if the negative sign is inside the parentheses, the negative sign will be affected by the exponent.

*Example:* 

\((-3)^3 = (-3)(-3)(-3) = -27\)

If the negative sign is inside the parentheses and the exponent is an even number, the answer will be positive. If the exponent is an odd number, then the answer will be negative.

*Examples:*

\(-2^3 = -8 \leftarrow \text{Since the negative sign is outside the parentheses, carry it out to the final answer.}\)

\((-5)^2 = (-5)(-5) = 25 \leftarrow \text{Since the negative sign is inside the parentheses, it needs to be carried out through the operation.}\)

\((-4)^2 = (-4)(-4) = 16 \leftarrow \text{Even number of exponents, positive answer.}\)

\((-4)^3 = (-4)(-4)(-4) = -64 \leftarrow \text{Odd number of exponents, negative answer.}\)
Negative Exponents – Whenever the problem, or the answer to the problem, contains negative exponents, these exponents need to be made positive. An answer with negative exponents will most likely be counted wrong. To change negative exponents into positive exponents, get the reciprocal fraction. In simpler words, if the negative exponent is on the top, move it down; if the negative exponent is on the bottom, move it up.

Examples:

\[ x^{-2} \quad \text{← Get the reciprocal, or move the negative exponent down.} \]

\[ x^{-2} = \frac{x^{-2}}{1} = \frac{1}{x^2} \]

\[ 5y^{-3} \quad \text{← Get the reciprocal of only the base with the negative exponent, the number stays in its place.} \]

\[ 5y^{-3} = \frac{5y^{-3}}{1} = \frac{5}{y^3} \]

\[ \frac{a^2}{b^{-4}} \quad \text{← Get the reciprocal of the base with the negative exponent, the base with the positive exponent stays in its place.} \]

\[ \frac{a^2}{b^{-4}} = \frac{a^2}{1} = a^2 \cdot b^4 \]
RADICALS

For every operation there is an opposite that can be used to cancel the operation. These opposites are called inverses.

Example: 

\[ + \text{ inverse } - \quad x^2 \text{ inverse } \sqrt{x} \]
\[ \times \text{ inverse } \div \quad x^3 \text{ inverse } \frac{1}{\sqrt[3]{x}} \]

The opposite/inverse of exponents is called a radical. If the exponent is a square, or 2, the inverse would be a square root; if the exponent is 3, the inverse would be a cubic root, and so on.

Examples:

\[ 2^2 = 4 \quad \text{inverse} \quad \sqrt{4} = 2 \]
\[ 2^3 = 8 \quad \text{inverse} \quad \sqrt[3]{8} = 2 \]
\[ 3^3 = 243 \quad \text{inverse} \quad \sqrt[3]{243} = 3 \]

Radicals can also be written in exponent notation, however, in this case the exponent would be a fraction.

Examples:

\[ a. \quad \sqrt[2]{4} = 4^{1/2} \quad \text{b.} \quad \sqrt[3]{8} = 8^{1/3} \quad \text{c.} \quad \sqrt[4]{243} = 243^{1/5} \]

The expression \( \sqrt[3]{8} \) is called a radical expression, where 3 is called the index, \( \sqrt{\text{ }} \) is the radical sign, and 8 is called the radicand. The index of a radical expression must always be a positive integer greater than 1. When no index is written it is assumed to be 2, or a square root; as noted in example ‘a’ above.

Negative Radicals – The only restriction that exists for negative signs and radicals is that there cannot be a negative sign under an even root since there is no real solution to this problem. However, you can have a negative sign in front of a radical or under odd roots and still be able to obtain a real number.

Examples:

\[ \sqrt{-16} \quad \text{<- Negative sign under even root, no real answer.} \]
\[ 4 \times 4 = 16 \quad \text{or} \quad -4 \times (-4) = 16 \]

\[ \sqrt[3]{-8} \quad \text{<- Negative sign under odd root, real answer.} \]
\[ -2 \times -2 \times -2 = -8 \]

\[ -\sqrt{81} \quad \text{<- Negative sign is outside the radical, the sign does not affect the calculations, it is only carried out to the final answer.} \]
\[ -\sqrt{81} = -(9)(9) = -81 \]
Radicals Containing Large Numbers – When working with large numbers under a radical sign, it is always easier to break down the number and work on a smaller part rather than trying to find an exact root.

Example:

\[
\sqrt{525} \quad \leftarrow \text{Break down the number into smaller numbers.}
\]

\[
\sqrt{21} \cdot \sqrt{25} \quad \leftarrow \text{Find the root of each of the smaller numbers.}
\]

\[
\sqrt{21} \cdot 5 \quad \leftarrow \text{Write final answer with single digits first and radicals at the end.}
\]

\[
5\sqrt{21}
\]

Sometimes it is not easy to find two small numbers that when multiplied give you the large numbers. In this case, you may break down the large number little by little.

Example:

\[
\sqrt{525} \quad \leftarrow \text{Follow the same procedure as described above. It takes a little longer, but at the end the same result is obtained.}
\]

\[
\sqrt{105} \cdot \sqrt{5}
\]

\[
\sqrt{21} \cdot \sqrt{25}
\]

\[
\sqrt{21} \cdot 5
\]

\[
5\sqrt{21}
\]
LAWS OF EXPONENTS AND RADICALS

1. \( x^m \cdot x^n = x^{m+n} \)
2. \( x^0 = 1 \) where \( x \neq 0 \)

3. \( x^{-n} = \frac{1}{x^n} \)
4. \( \frac{1}{x^{-n}} = x^n \)

5. \( \frac{x^m}{x^n} = x^{m-n} \)
6. \( \frac{x^m}{x^m} = 1 \)

7. \( (x^m)^n = x^{mn} \)
8. \( (xy)^n = x^n y^n \)

9. \( \left( \frac{x}{y} \right)^n = \frac{x^n}{y^n} \)
10. \( \left( \frac{x}{y} \right)^{-n} = \left( \frac{y}{x} \right)^n \)

11. \( \frac{1}{x^n} = \frac{1}{\sqrt[n]{x}} \)
12. \( x^{-\frac{1}{n}} = \frac{1}{\sqrt[n]{x}} \)

13. \( \sqrt[n]{x} \cdot \sqrt[n]{y} = \sqrt[n]{xy} \)
14. \( \frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}} \)

15. \( \sqrt[n]{\sqrt[n]{x}} = \sqrt[n]{x} \)
16. \( x^m = \sqrt[n]{x^m} = \left( \sqrt[n]{x} \right)^m \)

17. \( \left( \sqrt[n]{x} \right)^n = x \)

Note 1: The division of any number by zero is undefined. Also, if \( x \) is negative and \( n \) is even, \( \sqrt[n]{x} \) or \( x^\frac{1}{n} \) is not defined. Remember, this is like taking the even root of a negative number—\textit{not possible}.

Note 2: \( \frac{x}{0} \) is undefined \( 1^x = 1 \)
\( 0^x = 0 \) for \( x \neq 0 \) \( 0 \cdot x = 0 \)
\( 1 \cdot x = x \) \( \frac{x}{1} = x \)
EXPONENTS AND RADICALS - EXERCISES

Simplify:

1. \(x^5 \cdot x^7\)
2. \(x^0\) (for \(x \neq 0\))
3. \((a + b)^0\) (for \(a + b \neq 0\))

4. \(5^{-3}\)
5. \(\frac{2^2}{2^3}\)
6. \((3x)^2\)

7. \(\left(\frac{5}{2}\right)^{-2}\)
8. \(\left(\frac{5}{2}\right)^2\)
9. \((x^3)^2\)

10. \(\frac{x^2y}{xy}\)
11. \((3^2)^{-2}\)
12. \(9 \cdot 9^2\)

13. \(-\left(a^2\right)^4\)
14. \((-a^2)^4\)
15. \(\frac{c^7 \cdot c \cdot c^2}{c^5 \cdot c^2 \cdot c^3}\)

16. \(\left(a^2b^2c^2\right)\left(-ab\right)^4\)
17. \(\left(a^{-1}\right)^1\)
18. \(\left(4a^2b^2\right)\left(3a^2b^3\right)^3\)

19. \(\left(\frac{x^3}{x^2}\right)^3\)
20. \(\left(\frac{1}{k}\right)^{-2}\)
21. \(\left(a^7x^{-3}\right)^2\)

22. \(\frac{\left(a^2x^{-3}\right)^2}{\left(a^4b^{-1}\right)^3}\)
23. \((-1)^3\)
24. \((-1)^9\)

25. \(\left(\frac{8}{27}\right)^{\frac{2}{3}}\)
26. \(\sqrt[3]{27}\)
27. \(\sqrt[4]{72}\)

28. \(\sqrt[4]{16}\)
29. \(9^\frac{3}{2}\)
30. \(\left(\frac{1}{4}\right)^{\frac{7}{2}}\)

31. \(\sqrt{50}\)
32. \(\sqrt[3]{250}\)
EXPONENTS AND RADICALS - ANSWERS TO EXERCISES

1. \( x^5 \cdot x^7 = x^{12} \)

2. \( x^0 = 1 \)

3. \( (a + b)^0 = 1 \)

4. \( 5^{-3} = \frac{1}{5^3} = \frac{1}{125} \)

5. \( \frac{2^2}{2^3} = 2^{-1} = \frac{1}{2} \)

6. \( (3x)^2 = 3^2 \cdot x^2 = 9x^2 \)

7. \( \left( \frac{5}{2} \right)^{-2} = \left( \frac{2}{5} \right)^2 = \frac{2^2}{5^2} = \frac{4}{25} \)

8. \( \left( \frac{5}{2} \right)^2 = \frac{5^2}{2^2} = \frac{25}{4} \)

9. \( (x^3)^2 = x^{3(2)} = x^6 \)

10. \( \frac{x^2 y}{xy} = x^{(2-1)} \cdot y^{(1-1)} = x^1 \cdot y^0 = x \)

11. \( (3^2)^{-2} = 3^{-4} = \frac{1}{3^4} = \frac{1}{81} \)

12. \( 9 \cdot 9^2 = 9^3 \)

13. \( -(a^2)^4 = -(a^2)(a^2)(a^2)(a^2) = -a^8 \)

14. \( (-a^2)^4 = (-a^2)(-a^2)(-a^2)(-a^2) = a^8 \)

15. \( \frac{c^7 \cdot c \cdot c^2}{c^5 \cdot c^2 \cdot c^3} = \frac{c^{10}}{c^{10}} = 1 \)

16. \( (a^2 b^2 c^2)(-ab)^4 = a^{10} b^{10} c^6 \)

17. \( (a^{-1})^{-1} = a \)

18. \( (4a^2 b^2)(3a^2 b^3)^3 = 432a^{10} b^{11} \)

19. \( \left( \frac{x^3}{x^7} \right)^3 = \frac{x^9}{x^6} = x^3 \)

20. \( \left( \frac{1}{k} \right)^{-2} = \frac{k^2}{1^2} = k^2 \)

21. \( (a^2 x^{-3})^2 = a^4 x^{-6} = \frac{a^4}{x^6} \)

22. \( \frac{(a^2 x^{-3})^2}{(a^4 b^{-1})^2} = \frac{a^4 x^{-6}}{a^{12} b^{-3}} = \frac{b^3}{a^8 x^6} \)

23. \( (-1)^5 = -1 \)

24. \( (-1)^9 = -1 \)

25. \( \left( \frac{8}{27} \right)^{\frac{2}{3}} = \left( \frac{\sqrt[3]{8}}{\sqrt[3]{27}} \right)^2 = \left( \frac{2}{3} \right)^2 = \frac{9}{4} \)

26. \( \sqrt{27} = \sqrt{3 \cdot 9} = 3\sqrt{3} \)

27. \( \sqrt{72} = \sqrt{36 \cdot 2} = 6\sqrt{2} \)

28. \( \sqrt{16} = 2 \)

29. \( 9^{\frac{3}{2}} = \sqrt{9^3} = \sqrt{\sqrt{81} \cdot 9} = \sqrt{81} \cdot \sqrt{9} = 9 \cdot 3 = 27 \)

30. \( \left( \frac{1}{4} \right)^{\frac{7}{2}} = \sqrt[2]{\left( \frac{1}{4} \right)^7} = \left( \frac{1}{2} \right)^{\frac{7}{2}} = \frac{1}{128} \)

31. \( \sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2} \)

32. \( \sqrt{250} = \sqrt{125 \cdot 2} = 5\sqrt{2} \)
INTRODUCTION TO ALGEBRA

In Mathematics, you will often get expressions that contain one or more letters used to determine an unknown quantity. This letter is called a variable. The expression containing this variable is called a variable expression.

Examples: $x + 3$ ← In this case, the $x$ is the variable and the entire equation is the variable expression.

$3x + 4y$ ← This expression contains 2 variables, $x$ and $y$.

A variable expression is an expression that contains both variables and numbers:

Example: 

\[
5x + y + 12 + 4
\]

variable terms  constant terms

In fact: $x + y$

is also a variable expression because even though not seen, there is a one in front of both the $x$ & $y$ variable: $1x + 1y$.

Evaluating a variable expression means replacing the variable with a given number. After plugging in the numbers given, the variable expression is reduced to a single number containing no letters or variables.

Example: Evaluate $2x + y$ when $x = 2$ and $y = 3$

$2x + y$

$2(2) + 3$

$4 + 3 = 7$

Not all the time will you get variable expressions that contain only one or two variables. Variable expressions may also contain numerous terms, which can be combined to simplify the equation. To simplify an expression combine “like terms,” or terms that contain the exact same variables and exponent if applicable.

Example:  $9xy + 3 - 10x + 5x - 7xy + 4$ ← In this expression we have two variable terms, $x$ and $xy$, and two constant terms, 3 and 4.

To simplify variable expressions, combine like terms together by adding their coefficient values. The coefficient is the number in front of the variable. In the example below: 9, -7, -10 and 5 are coefficients. Notice that: 3 and 4 are not; these are not attached to any variables.

Example:  $9xy - 7xy - 10x + 5x + 3 + 4$ ← Combine $xy$ with $xy$: $9xy - 7xy = 2xy$

$= 2xy - 5x + 7$

$x$ with $x$: $-10x + 5x = -5x$

and numbers with numbers: $3 + 4 = 7$

Note: coefficients are the numbers along with their sign: in the example above:
-7 is the coefficient of $xy$
-10 is the coefficient of $x$
1. Evaluate $6x^2 + 5xy - z$ when $x = -1$, $y = 4$, and $z = -3$

2. Evaluate $2x^2 + 6y - 3z$ when $x = -5$, $y = 7$, and $z = 3$

3. Evaluate $8x^3 + 8xy - 2z$ when $x = -7$, $y = 4$, and $z = -5$

4. Simplify $9xy - 16x + 7x - 5xy$

5. Simplify $15xy - 25x + 9x - 2xy$

6. Simplify $8(3x + 5) - 8x - 5$
INTRODUCTION TO ALGEBRA - ANSWERS TO EXERCISES

1. \[6x^2 + 5xy - z = 6(-1)^2 + 5(-1)(4) - (-3) = 6 - 20 + 3 = -11\]

2. \[2x^2 + 6y - 3z = 2(-5)^2 + 6(7) - 3(3) = 50 + 42 - 9 = 83\]

3. \[8x^3 + 8xy - 2z = 8(-7)^3 + 8(-7)(4) - 2(-5) = -2744 - 224 + 10 = -2958\]

4. \[9xy - 16x + 7x - 5xy = (9 - 5)xy + (-16 + 7)x = 4xy - 9x\]

5. \[15xy - 25x + 9x - 2xy = (15 - 2)xy + (-25 + 9)x = 13xy - 16x\]

6. \[8(3x + 5) - 8x - 5 = 24x + 40 - 8x - 5 = (24 - 8)x + (40 - 5) = 16x + 35\]
LINEAR EQUATIONS

A linear equation can be defined as an equation in which the highest exponent of the equation variable is one. When graphed, the equation is shown as a single line.

Example: \[ x + 2 = 4 \quad \leftrightarrow \quad \text{Linear equation: highest exponent of the variable is 1.} \]

A linear equation has only one solution. The solution of a linear equation is equal to the value of the unknown variable that makes the linear equation true.

Example: \[ x + 2 = 4 \quad \leftrightarrow \quad \text{The value of the unknown variable that makes the equation true is 2:} \quad 2 + 2 = 4 \]

There are different forms of writing a linear equation. Each form has a different way of solving the linear equation that makes the process easier. There is only one rule that applies to any form of linear equations: whatever you do on one side you need to do on the other side. That is, if you add on the left side, you need to add on the right side. If you multiply on the left side, you need to multiply on the right side, and so on.

1. Equations of the form \( x + a = b \) or \( x - a = b \).

Linear equations in this form are solved by adding or subtracting the same quantity to both sides with the idea of leaving the variable by itself. That is, isolating the variable.

Examples:
\[
\begin{align*}
x - 5 &= 10 \\
x - 5 + 5 &= 10 + 5 \\
x &= 15
\end{align*}
\]
\[ \leftrightarrow \quad \text{Add 5 to both sides to eliminate the number on the left side of the equation and leave } x \text{ alone.} \]

\[
\begin{align*}
y + 3.2 &= 4.1 \\
y + 3.2 - 3.2 &= 4.1 - 3.2 \\
y &= 0.9
\end{align*}
\]
\[ \leftrightarrow \quad \text{Subtract 3.2 to both sides to eliminate the number on the left side and isolate } y. \]

2. Equations of the form \( ax = b \).

Linear equations in the form of multiplication are solved by dividing both sides of the equation by the number multiplying the variable. When a fraction is multiplying the variable, multiply both sides of the equation by the reciprocal of the fraction attached to the variable.
Examples: \[3x = 18\] 
\[\frac{3}{3} x = \frac{18}{3}\] \[x = 6\]  

\[
\frac{3}{4} x = \frac{2}{5} \\
\frac{4}{3} \cdot \frac{3}{4} x = \frac{2}{5} \cdot \frac{4}{3} \\
x = \frac{8}{15}
\]

← Divide both sides by 3 to leave \(x\) by itself.

← Multiply both sides by the reciprocal of \(\frac{3}{4}\), which is \(\frac{4}{3}\).

3. **Equations of the form** \(ax + b = c\).

This type of linear equation presents a combination of the first two forms of linear equations. The first step in solving this form of linear equations is to eliminate constant (the number unattached to any variable). To do this add or subtract the same quantity to both sides of the equation. Next, eliminate the number attached to the variable by dividing or multiplying, depending on the case, to both sides of the equation. The idea is to leave variables on one side of the equal sign and constants on the other side of the equal sign.

Examples: \[3y + 7 = 25\] 
\[3y + 7 - 7 = 25 - 7\] \[3y = 18\] 
\[\frac{3}{3} y = \frac{18}{3}\] \[y = 6\]  

\[
\frac{8}{5} x + 2 = -1 \\
\frac{8}{5} x + 2 - 2 = -1 - 2 \\
\frac{8}{5} x = -3 \\
\frac{5}{8} \cdot \frac{8}{5} x = -3 \cdot \frac{5}{8} \\
x = -\frac{15}{8}
\]

← Subtract 7 from both sides to remove it from the left side.

← Divide both sides by 3 to leave \(y\) alone and solve for \(y\).

← Follow the steps described above to solve for \(x\).
4. Equations of the form $ax + b = cx + d$.

In this form of linear equations the main goal is to leave the variables on one side and the constants on the other side. To do this, perform all the steps required to solve the previous forms of linear equations step by step. Do not try to do all the steps at once, this can only result in confusion and mistakes. Be careful!

Example: \[6x - 15 = -8x + 13\] \<– Start by moving the variable from the right side to the left side.\]
\[6x + 8x - 15 = -8x + 8x + 13\] \<– Combine the two variables together.\]
\[14x - 15 = 13\] \<– Add 15 to both sides to eliminate the constant on the left side of the equation.\]
\[14x - 15 + 15 = 13 + 15\]
\[14x = 28\] \<– Divide both sides of the equation by 14.\]
\[
\begin{align*}
14 & \quad x = 28 \\
14 & \quad 14 \\
\frac{14}{14} & \quad x = 2
\end{align*}
\]<– Solve for \(x\).\]

Distributive Property: \[a(b + c) = ab + ac\]

The distributive property is used to remove grouping symbols in linear equations. Grouping symbols are terms enclosed in parentheses. To ungroup, distribute the term outside the parentheses to each of the terms inside the parentheses by multiplication.

Example: \[2(x - 3) + 1 = -7\] \<– Multiply the term outside the parentheses to each of the terms inside the parentheses.\]
\[2x - 6 + 1 = -7\] \<– Eliminate the constant on the left side of the equation by adding 5 to both sides of the equation.\]
\[2x - 5 = -7 + 5\]
\[2x = -2\]
\[
\begin{align*}
\frac{2}{2} & \quad x = \frac{-2}{2} \\
x & \quad = -1
\end{align*}
\]<– Divide both sides by 2 and solve for \(x\).\]
Solve for x:

1. \(8x - 6 = 12\)
2. \(4x - 5 = 2x + 7\)
3. \(x - 1 - 3(5 - 2x) = 2(2x - 5)\)
4. \(\frac{2x - 4}{4} = \frac{4x + 3}{3}\)
5. \(\frac{2x - a}{b} = \frac{4x - b}{a}\)
6. \(\frac{3}{x + 1} - \frac{4}{5} = \frac{1}{x + 1} - 6\)
7. \(\frac{3}{x - 1} + \frac{4}{x + 1} = \frac{8}{x + 1}\)
8. \(\frac{1}{ax} + \frac{1}{bx} = \frac{1}{c}\)
9. \(\frac{3 - 4}{x} = 10\)
10. \(\frac{2x + 1}{5} + \frac{x - 4}{6} = \frac{2}{3}\)
11. \(3(2x - 1) + x = 5x + 3\)
12. \(3 - 5(2x - 5) = -2\)
13. \(-3(5x + 7) - 4 = -10x\)
14. \(8 + 3(x + 2) = 5(x - 1)\)
15. \(5(x + 2) - 4(x + 1) = 7\)
16. \(\frac{1}{2x + 1} - \frac{3}{4} = \frac{7}{2x + 1}\)
17. \(\frac{7}{2x - 3} + \frac{4}{2x - 3} = 6\)
LINEAR EQUATIONS – ANSWERS TO EXERCISES

1. \(8x - 6 = 12\)
   \[8x - 6 + 6 = 12 + 6\]
   \[8x = 18\]
   \[x = \frac{18}{8} = \frac{9}{4}\]

2. \(4x - 5 = 2x + 7\)
   \[4x - 5 - 2x = 2x + 7 - 2x\]
   \[2x - 5 = 7\]
   \[2x = 12\]
   \[x = 6\]

3. \(x - 1 - 3(5 - 2x) = 2(2x - 5)\)
   \[x - 1 - 15 + 6x = 4x - 10\]
   \[7x - 4x = -10 + 15\]
   \[3x = 5\]
   \[x = \frac{5}{3}\]

4. \(2x - 4 = 4x + 3\)
   \[\frac{2x - 4}{4} = \frac{4x + 3}{4}\]
   \[3(2x - 4) = 4(4x + 3)\]
   \[6x - 12 = 16x + 12\]
   \[6x - 16x = 12 + 12\]
   \[-10x = 24\]
   \[x = -\frac{12}{10} = -\frac{6}{5}\]

5. \(2x - a = 4x - b\)
   \[\frac{2x - a}{b} = \frac{4x - b}{a}\]
   \[a(2x - a) = b(4x - b)\]
   \[2ax - a^2 = 4bx - b^2\]
   \[2ax - 4bx = a^2 - b^2\]
   \[x(2a - 4b) = a^2 - b^2\]
   \[x = \frac{a^2 - b^2}{2a - 4b}\]

6. \(\frac{3}{x+1} - \frac{4}{5} = \frac{1}{x+1} - 6\)
   \[5(x+1)\left(\frac{3}{x+1} - \frac{4}{5}\right) = 5(x+1)\left(\frac{1}{x+1} - 6\right)\]
   \[15 - 4(x+1) = 5 - 30(x+1)\]
   \[15 - 4x - 4 = 5 - 30x - 30\]
   \[26x = -36\]
   \[x = -\frac{18}{13}\]

7. \(\frac{3}{x-1} + \frac{4}{x+1} = \frac{8}{x+1}\)
   \[(x-1)(x+1)\left(\frac{3}{x-1} + \frac{4}{x+1}\right) = (x+1)\left(\frac{8}{x+1}\right)\]
   \[3(x+1) + 4(x-1) = 8(x-1)\]
   \[3x + 3 + 4x - 4 = 8x - 8\]
   \[7x - 8x = 1 - 8\]
   \[-x = -7\]
   \[x = 7\]

8. \(\frac{1}{ax} + \frac{1}{bx} = \frac{1}{c}\)
   \[abc\left(\frac{1}{ax} + \frac{1}{bx}\right) = abcx\left(\frac{1}{c}\right)\]
   \[bc + ac = abx\]
   \[bc + ac = x\]
   \[\frac{ab}{x} = \frac{c(a+b)}{ab}\]
   \[x = \frac{ab}{c(a+b)}\]

9. \(\frac{3}{x} - \frac{4}{3x} = 10\)
   \[3x\left(\frac{3}{x} - \frac{4}{3x}\right) = 3x(10)\]
   \[9 - 4 = 30x\]
   \[x = \frac{1}{6}\]
10. \[
\frac{2x+1}{5} + \frac{x-4}{6} = \frac{2}{3}
\]
\[
30 \left( \frac{2x+1}{5} + \frac{x-4}{6} \right) = 30 \left( \frac{2}{3} \right)
\]
\[
6(2x+1)+5(x-4) = 20
\]
\[
12x+6+5x-20 = 20
\]
\[
17x = 34
\]
\[
x = 2
\]

11. \[
3(2x-1)+x = 5x+3
\]
\[
6x-3 + x = 5x+3
\]
\[
7x-5x = 3+3
\]
\[
2x = 6
\]
\[
x = 3
\]

12. \[
3 - 5(2x-5) = -2
\]
\[
3-10x+25 = -2
\]
\[
-10x = -2 - 3 - 25
\]
\[
-10x = -30
\]
\[
x = 3
\]

13. \[
-3(5x+7) - 4 = -10x
\]
\[
-15x - 21 - 4 = -10x
\]
\[
-15x + 10x = 21 + 4
\]
\[
-5x = 25
\]
\[
x = -5
\]

14. \[
8 + 3(x+2) = 5(x-1)
\]
\[
8 + 3x + 6 = 5x - 5
\]
\[
3x - 5x = -5 - 8 - 6
\]
\[
-2x = -19
\]
\[
x = \frac{19}{2}
\]

15. \[
5(x+2) - 4(x+1) = 7
\]
\[
5x + 10 - 4x - 4 = 7
\]
\[
x = 7 - 10 + 4
\]
\[
x = 1
\]

16. \[
\frac{1}{2x+1} - \frac{3}{4} = \frac{7}{2x+1}
\]
\[
4(2x+1) \left( \frac{1}{2x+1} - \frac{3}{4} \right) = 4(2x+1) \left( \frac{7}{2x+1} \right)
\]
\[
4 - 3(2x+1) = 28
\]
\[
4 - 6x - 3 = 28
\]
\[
-6x - 3 = 24
\]
\[
x = -\frac{27}{6}
\]

17. \[
\frac{7}{2x-3} + \frac{4}{2x-3} = 6
\]
\[
(2x-3) \left( \frac{7}{2x-3} + \frac{4}{2x-3} \right) = (2x-3) \cdot 6
\]
\[
7 + 4 = 6(2x-3)
\]
\[
11 = 12x - 18
\]
\[
11 + 18 = 12x
\]
\[
x = \frac{29}{12}
\]
WORD PROBLEMS

Word problems are mathematical problems or equations expressed in words rather than numbers. The main idea of word problems is to try to understand and translate what is being said in words into numbers and variables, which are the unknowns. Solving word problems begins with a good translation. If you can successfully understand and are able to translate words into numerical relations, word problems will not be difficult. As with any mathematical problem, the key to success is practice.

Translating Words into Mathematical Problems

When converting word problems to equations, certain "key" words tell you what kind of operations to use: addition, multiplication, subtraction, and division. The table below shows some common phrases and the operation to use.

<table>
<thead>
<tr>
<th>Word</th>
<th>Operation</th>
<th>Example</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum</td>
<td>Addition</td>
<td>The sum of my age and 10 equals 27.</td>
<td>( x + 10 = 27 )</td>
</tr>
<tr>
<td>Total</td>
<td>Addition</td>
<td>The total of my pocket change and 20 dollars is $22.43.</td>
<td>( x + 20 = 22.43 )</td>
</tr>
<tr>
<td>More Than</td>
<td>Addition</td>
<td>Eleven more than my age equals 43.</td>
<td>( 11 + x = 43 )</td>
</tr>
<tr>
<td>Difference*</td>
<td>Subtraction</td>
<td>The difference between my age and my younger sister's age, which is 11 years old, is 5 years.</td>
<td>( x - 11 = 5 )</td>
</tr>
<tr>
<td>Less Than*</td>
<td>Subtraction</td>
<td>Seven less than my age equals 32.</td>
<td>( x - 7 = 32 )</td>
</tr>
<tr>
<td>Times</td>
<td>Multiplication</td>
<td>Three times my age is 60.</td>
<td>( 3 \times y = 60 )</td>
</tr>
<tr>
<td>Product</td>
<td>Multiplication</td>
<td>The product of my age and 14 is 168.</td>
<td>( y \times 14 = 168 )</td>
</tr>
</tbody>
</table>

*Note: When the problem asks for the difference of two numbers, the greater number should always go first to ensure you obtain a positive number.

Algebra Dictionary

<table>
<thead>
<tr>
<th>Words</th>
<th>Mathematical Way to Say It</th>
</tr>
</thead>
<tbody>
<tr>
<td>is, are, was, has, cost, is equal to</td>
<td>=</td>
</tr>
<tr>
<td>is not equal to</td>
<td>≠</td>
</tr>
<tr>
<td>which, what</td>
<td>( y ) (or any other letter)</td>
</tr>
<tr>
<td>and, more, sum, older than</td>
<td>+ (addition)</td>
</tr>
<tr>
<td>less, difference, younger than</td>
<td>− (subtraction)</td>
</tr>
<tr>
<td>times, product</td>
<td>( \times ) (multiplication)</td>
</tr>
<tr>
<td>quotient of ( y ) and ( z ), or ( y ) divided by ( z )</td>
<td>( \frac{y}{z} ) or ( y \div z )</td>
</tr>
<tr>
<td>greater than</td>
<td>&gt;</td>
</tr>
<tr>
<td>less than</td>
<td>&lt;</td>
</tr>
<tr>
<td>the square of ( x )</td>
<td>( x^2 )</td>
</tr>
<tr>
<td>( c ) times as old as John</td>
<td>( (c) \times ) (John's age)</td>
</tr>
</tbody>
</table>
Translate these phrases into correct mathematical expressions:

1. $y$ increased by four
2. the sum of $m$ and $n$
3. the product of $k$ and $w$
4. $s$ divided by $t$
5. the square of $x$
6. seven multiplied by $p$
7. two-thirds of $y$
8. $a$ less $b$
9. the sum of four $y$’s
10. the difference between $x$ and $y$
11. the quotient of $3a$ and $4b$
12. five decreased by $e$
13. three times the product of $m$ and $n$
14. twice $x$ divided by four
15. one-half increased by the product of three and $x$
16. three times the square of $y$
17. the square of (three multiplied by $a$)
18. the product of $h$ and the square of $k$
19. a number increased by 8
20. eight less than some number $t$
21. the difference between a number $y$ and a number $x$
22. seven diminished by a number $x$
23. the square of the sum of $x$ and four
24. the sum of the squares of $x$ and four
25. four added to twice an unknown quantity $x$
26. twice an unknown quantity, $x$, diminished by seven
27. the square of $r$ minus the product of $x$ and $y$
28. twice the product of the square of $x$ and the square of $y$
29. the quotient of five times the square of $w$ and four times the cube of $y$
30. the difference between the cube of $x$ and the square of $z$
1. $y + 4$
2. $m + n$
3. $k \cdot w$
4. $\frac{s}{t}$
5. $x^2$
6. $7p$
7. $\frac{2}{3}y$
8. $a - b$
9. $y + y + y + y = 4y$
10. $x - y$
11. $\frac{3a}{4b}$
12. $5 - e$
13. $3 \cdot m \cdot n$
14. $\frac{2x}{4}$
15. $\frac{1}{2} + 3x$
16. $3y^2$
17. $(3a)^2$
18. $h \cdot k^2$
19. $x + 8$
20. $t - 8$
21. $y - x$
22. $7 - x$
23. $(x + 4)^2$
24. $x^2 + 4^2$
25. $2x + 4$
26. $2x - 7$
27. $r^2 - xy$
28. $2x^2y^2$
29. $\frac{5w^2}{4y^3}$
30. $x^3 - z^2$
SOLVING WORD PROBLEMS

Word problems can be classified into different categories. Understanding each category will give you an advantage when trying to solve word problems. All problems in each category are solved the same way. With practice you will begin to see the similarities among word problems and be able to solve them easier and faster.

Age Problems

These types of problems ask you to figure out the age of different people by giving you different clues.

Example: John is 3 years older than Jim. Jim is 4 years less than twice David’s age. How old are the three boys if their ages add up to 35?

Let:  
David’s age = \( x \)  
Jim’s age = \( 2x - 4 \)  
John’s age = \( (2x - 4) + 3 \)

\[ \text{Jim’s age} = x + 2x - 4 + (2x - 4) + 3 = 35 \]  
\[ 5x = 40 \]  
\[ x = 8 \]  
\[ 2x - 4 = 12 \]  
\[ (2x - 4) + 3 = 15 \]  

David’s age  
Jim’s age  
John’s age

Coin Problems

These types of problems involve figuring out the number of coins per given value.

Example: A cashier has 3 more dimes than nickels and twice as many nickels as quarters. Find the number of each kind of coin if the total value of the coins is $3.05.

Let:  
# of quarters = \( x \)  
# of nickels = \( 2x \)  
# of dimes = \( 2x + 3 \)

\[ 0.25x + 0.05(2x) + 0.10(2x + 3) = 3.05 \]  
\[ 0.25x + 0.10x + 0.20x + 0.30 = 3.05 \]  
\[ 0.55x = 2.75 \]  
\[ x = 5 \]  
\[ 2x = 10 \]  
\[ 2x + 3 = 13 \]  

Total # of quarters  
Total # of nickels  
Total # of dimes
Distance Problems

**Example:** You are driving along at 55 mph when you are passed by a car doing 85 mph. How long will it take for the car that passed you to be one mile ahead of you?

Let $D$ be the distance that you travel in time $t$, and $D + 1$ be your distance plus one mile ahead of you that the other car traveled in time $t$.

Using the rate equation in the form $distance = speed \cdot time$, or $D = s \cdot t$ for each car, we can write:

$D = 55t$ and $D + 1 = 85t$

Substituting the first equation into the second $\Rightarrow 55t + 1 = 85t$

and solving for $t \Rightarrow -30t = -1$

$t = 1/30$ hours or 2 minutes

Geometry Problems

**Example:** If the perimeter of a rectangle is 18 inches, and one side is one inch longer than the other, how long are the sides?

Let one side be $x$ (width) and the other side be $x + 1$ (length).

The perimeter of a rectangle is found by the formula: $P = 2w + 2l$

Then the given condition may be expressed as:

$2x + 2(x + 1) = 18$

$x + 2x + 2 = 18$

$4x + 2 = 18$

$x = 4$ Therefore, the sides have length 4 and 5.

Investment Problems

**Example:** Suppose $10,000 is invested at 9% interest. How much money must be invested at 12% to produce a return of 11% on the entire amount invested?

Let: amount invested at 12% = $x$

amount invested at 9% = 10,000

amount invested at 11% = $x + 10,000$

$0.12x + 0.09(10,000) = 0.11(x + 10,000)$

$12x + 9(10,000) = 11(x + 10,000) \quad \Rightarrow$ after multiplying both sides by ‘10’.

$12x + 90,000 = 11x + 110,000$

$x = $20,000
Mixture Problems

Example: The instructions on a can of powdered drink mix say to mix 1/4 cup of the mix with 2 quarts of water. How much of the mix should be used with 1 1/2 gallons of water?

Let \( x \) = # of cups of the drink mix to use

\[
\frac{1}{4} \times 2 = \frac{x}{6} \quad \text{There are 6 quarts in 1 ½ gallons}
\]

\[
6 \left( \frac{1}{4} \right) = 2x
\]

\[
\frac{3}{2} = 2x
\]

\[
x = \frac{3}{4} \text{ cup of the drink mix}
\]

Number Problems

Example: Find a number such that 5 more than one-half the number is three times the number.

Translating into math: \( 5 + \frac{x}{2} = 3x \) \( \leftarrow \) let \( x \) be the unknown number.

Solving: \( 2 \left( 5 + \frac{x}{2} \right) = 2(3x) \) \( \rightarrow \) \( 10 + x = 6x \)

\[
10 = 5x
\]

\[
x = 2
\]

Percent of Problems

Example: The price of gasoline increased by 25% between January and March. If the price per gallon in March was $1.15, what was the price per gallon in January?

To find the price in March: \( \text{Price in January} + 25\% \text{ increase in cost} = \text{Price in March} \)

Let: \( \text{price per gallon in January} = x \)

\( 25\% \text{ increase in cost} = 0.25x \)

\( \text{price in March} = 1.15 \)

Then \( x + 0.25x = 1.15 \)

\( 1.25x = 1.15 \)

\( x = \$0.92 \)
Work Problems

Example: A fast employee can assemble 7 radios in an hour, and another slower employee can only assemble 5 radios per hour. If both employees work together, how long will it take to assemble 26 radios?

The two together will build 7 + 5 = 12 radios in an hour, so their combined rate is 12 radios per hour.

Using \( \text{Quantity} \) \( \frac{\text{Time}}{\text{Rate}} = \frac{26}{12} = 2 \frac{1}{6} \) hour \( \rightarrow \) 2 hours 10 minutes

Since many of the word problems can be solved in a similar way, the following lists a series of recommended steps to follow when solving word problems which will make the problem easier to solve.

List of Steps to Follow When Solving Word Problems

1. Read the problem carefully to determine what the problem is saying and what it is asking for. Read it as many times as necessary to understand it.

2. Read the problem again and write down details.
   
   A. Determine what question is being asked.
      
      i. **Key words** – to determine what unknown quantity you are asked to find, look for key words such as how many, how much, what is, find, how long.
      
      ii. **Associated words** – find the word or words associated with the key words.

Examples: (Key words are underlined, associated words are in italics)

- How many papers did he sell?
- How much money was left?
- What are the lengths of the two bars?
- Find the dimensions of the rectangle.
- How long will it take for John to save $200?
- Determine the percentage increase in the price per unit.

iii. For complicated problems, it may help to write down in your own words what question is being asked.
B. Write down essential details.

   i. Make diagrams, charts, or drawings to help you visualize the problem.

   ii. Assign a letter to represent the unknown quantity (or one of the unknowns if there is more than one) and write down the letter and what it represents.

     Example: Let $x =$ cost per unit

   iii. Determine what units the solution should have and write this down.

   iv. If there is more than one unknown quantity to find, establish and write down the relationships among the unknowns.

     Example: Let $x =$ Pat’s age, then $2x - 3 =$ Laura’s age

     If there is no such relationship, assign other letter(s) to represent the other unknown(s). Remember that to find a unique solution you must be able to write down as many equations as you have letters representing unknowns.

   v. If there are several known quantities given in the problem, it may be helpful to write them down in tabular form.

     Example:

     |                |                |
     |----------------|----------------|
     | Amount of loan | $10,000        |
     | Interest rate  | 9%             |
     | Monthly payments | $200 per month |

3. If still unclear as to what is being asked, or can’t find the relationships among the data given, read the problem again. In some word problems, relationships are straightforward and easy to find. Other problems may require several readings before the relationships become clear. In the latter case, a chain of deductive reasoning may be required to link the data given in the problem to the problem solution. Do not get discouraged. Usually with each reading you will understand the problem a little better.

4. Write down the equation or inequality which expresses the relationships among the unknowns and all other relevant quantities in the problem.

5. Solve the equation or inequality. Identify the unknowns in the problem using the solution you obtained.

6. Check your results.
WORD PROBLEMS – EXERCISES

1. One number is 6 less than 3 times another number and their sum is 62. Find the numbers.

2. In a basketball game, the number of points scored by the Miners was equal to 20 less than twice their opponent’s score. The total number of points scored was 127. What was the total score?

3. What is the velocity of a car (in miles per hour) if it goes 40 miles in 35 minutes?

4. Carter sells cashews for $3.00 a pound, hazelnuts for $2.50 a pound, and peanuts for $1.75 a pound. How many pounds of cashews and hazelnuts should be mixed with 50 lbs of peanuts to obtain a mixture of 100 lbs that will sell for $2.30 a pound so that the profit or loss is unchanged?

5. One solution is 10% acid and another is 4% acid. How many cubic centimeters (cc) of each should be mixed to obtain 150 cc of a solution that is 6% acid?

6. A company manufactures its product at a cost of 50 cents per item and sells it for 85 cents per item. Daily overhead is $600. How many items must be manufactured each day in order for the company to break even?

7. Jim is 3 times as old as his cousin and the difference in their ages is 18. How old is Jim?

8. Maria has scores of 96, 86, and 78 on three tests. What must her average score on the next two tests be in order for her to have an average of at least 90?

9. What are the dimensions of a rectangle whose length is 4 more than twice the width and whose perimeter is 3 less than 7 times the width?

10. A carpenter cuts a board into three pieces of equal length and then cuts off ¼ of one of the pieces. If the smallest board he has is 1 foot in length, what was the length of the original board?

11. Janet and Susan together earned $109.50 in a week. Janet worked 7 hrs and Susan worked 9.5 hrs. If each had worked three hours more, their combined pay would have been $150. What is the hourly rate for each?

12. Last year Eduardo invested some money at 8% interest and his wife, Sofia, invested some money at 9% interest. Their total interest on the two investments was $860. This year they each decided to invest the same amount of money as last year. Eduardo invested his at 7% and Sofia invested hers at 6%. If they receive a total of $640 interest, how much money did each invest?

13. Jennifer got a job as an engineer at a starting salary of $28,000. Miguel got a job as an accountant at a starting salary of $24,000. Jennifer will receive an annual increase of $600 and Miguel’s annual increase will be $1100. In how many years will their salaries be equal?

14. David can paint the living room in 4 hours and Anna can paint it in 6 hours. How long will it take the two of them to paint the living room if they work together?

15. A restaurant uses 2 pints of milk with 3 pints of cream to make coffee creamier. To make a large quantity of the mixture, how many pints of cream should be used with 18 pints of milk?
1. Let \( x \) = one numbers
   \( y \) = another number
   \( x = 3y - 6 \)
   \( x + y = 62 \)
   \( y + 3y - 6 = 62 \)
   \( 4y = 68 \)
   \( y = 17 \)
   \( 3y - 6 = 45 \)  The two numbers are 17 and 45

2. Let \( x \) = the opponent’s score
   Miner's score = \( 2x - 20 \)
   \( x + 2x - 20 = 127 \)
   \( 3x = 147 \)
   \( x = 49 \)  (opponent's score)
   \( 2x - 20 = 78 \)  (Miner's score)

3. \( \frac{40}{35} = \frac{x}{60} \)
   \( 35x = 2400 \)
   \( x = 68.57 \) miles/hour

4. Let \( x \) = # of lbs of cashews
   # of lbs of hazelnuts = \( 50 - x \)
   \( 3x + 2.5(50 - x) + 1.75(50) = 2.30(100) \)
   \( 3x + 125 - 2.5x + 87.5 = 230 \)
   \( 0.5x = 17.5 \)
   \( x = 35 \) lbs of cashews
   \( 50 - x = 15 \) lbs of hazelnuts

5. Let \( x \) = amount of 10% solution
   Amount of 4% solution = \( 150 - x \)
   \( 0.10x + 0.04(150 - x) = 0.06(150) \)
   \( 10x + 4(150 - x) = 6(150) \)
   \( x = 50 \) cc of 10% solution
   \( 150 - x = 100 \) cc of 4% solution
6. Let $x =$ # of items
   
   $0.50x + 600 = 0.85x$
   
   $50x + 60000 = 85x$
   
   $x = 1715$

7. Let $x =$ cousin's age
   
   Jim's age = $3x$
   
   $3x - x = 18$
   
   $x = 9$ (cousin's age)
   
   $3x = 27$ (Jim's age)

8. Let $x =$ average score on the next two tests
   
   $\frac{96 + 86 + 78 + 2x}{5} \geq 90$
   
   $260 + 2x \geq 450$
   
   $x \geq 95$

9. Let $x =$ width of the rectangle
   
   Length of the rectangle = $2x + 4$
   
   Perimeter of the rectangle = $7x - 3$
   
   $2x + 2(2x + 4) = 7x - 3$
   
   $x = 11$ units (width)
   
   $2x + 4 = 26$ units (length)
   
   $7x - 3 = 74$ units (perimeter)

10. Let $x =$ length of original board
    
    $\frac{1}{4}\left(\frac{x}{3}\right) = 1$
    
    $x = 12$ feet

11. Let $x =$ Janet's rate
    
    Susan's rate = $y$
    
    $7x + 9.5y = 109.5$
    
    $10x + 12.5y = 150$
    
    $70x + 95y = 1095$
    
    $-70x - 87.5y = -1050$
    
    $7.5y = 45$
    
    $y = $6.00
    
    $7x + 9.5(6) = 109.5$
    
    $x = $7.50  Janet's rate is $7.50 per hour and Susan's rate is $6.00 per hour
12. Let $x$ = amount Eduardo invested
   Amount Sofia invested = $y$
   
   $0.08x + 0.09y = 860$
   $8x + 9y = 86000$
   $16x + 18y = 172000$
   
   $0.07x + 0.06y = 640$
   $7x + 6y = 64000$
   $-21x - 18y = -192000$
   
   $-5x = -20000$

   $7x + 6y = 64000$

   $x = $4000

   $7(4000) + 6y = 64000$

   $28000 + 6y = 64000$

   Eduardo invested $4000

   $y = $6000

   Sofia invested $6000

13. Let $x$ = # of years until salaries are equal
    $28000 + 600x = 24000 + 1100x$

    $-500x = -4000$

    $x = 8$ years until their salaries are equal

14. Let $x$ = # of hours
    
    $\frac{x}{4} + \frac{x}{6} = 1$

    $3x + 2x = 12$

    $x = 2.4$ hours

15. Let $x$ = # of pints of cream
    
    $\frac{2}{3} = \frac{18}{x}$

    $2x = 54$

    $x = 27$ pints of cream