**EXPONENTS**

Exponents are used to write long multiplications in a short way. The exponent will tell you how many times the number or variable needs to be multiplied. In this case the variable is ‘a’.

Examples:

\[(6)(6) = 6^2 \quad a \cdot a \cdot a = a^3\]

In exponential notation, the number or variable being multiplied several times is called the **base**. The exponent, or number that tells you how many times you need to multiply, is called the **power**.

\[2^3 \quad \rightarrow \quad 2 \text{ is the base, and } 3 \text{ is the power}\]

Exponents are mostly used when dealing with variables, or letters, since it is easier and simpler to write \(x^4\) than \(x \cdot x \cdot x \cdot x\). Also, letters are used for they represent an unknown number, and so, the term: **variable**.

There are a few rules used for simplifying exponents:

**Zero Exponent Rule** – Any number or letter raised to the zero power is always equal to 1.

*Example:* \(3^0 = 1\) \(a^0 = 1\)

**Product Rule** – When multiplying the same base, the exponents are added together.

*Example:* \(x^3 \cdot x^4 \quad \leftarrow \text{Same base, } x, \text{ add the exponents, } 3 + 4 = 7\)

\[x^3 \cdot x^4 = x^7\]

**Quotient Rule** – When dividing the same base, subtract the exponents.

*Example:* \(\frac{x^5}{x^2} \quad \leftarrow \text{Same base, } x, \text{ subtract the exponents, } 5 - 2 = 3\)

\[\frac{x^5}{x^2} = x^3\]
**Power Rule** – When the operation contains parentheses, multiply the exponent in the inside with the exponent on the outside.

*Example:* \((y^6)^2 \leftarrow \text{Multiply the exponents, } 6 \times 2 = 12\)

\((y^6)^2 = y^{12}\)

When there is a fraction inside the parentheses, the exponent multiplies on the current power of the numerator and the denominator. However, this rule does not apply if you have a sum or difference within the parentheses; in that case a different rule will apply.

*Examples:*  
\[
\left( \frac{3}{4} \right)^2 = \left( \frac{3}{4} \right)^2 = \frac{9}{16}
\]
\[
\left( \frac{x^3}{y^2} \right)^4 = \left( \frac{x^3}{y^2} \right)^4 = \frac{x^{12}}{y^8}
\]

Be careful: \((\frac{a+b}{c+d})^2\) is not the same as \(\frac{a^2+b^2}{c^2+d^2}\).

In fact: \[
\left( \frac{a+b}{c+d} \right)^2 = \frac{a^2 + 2ab + b^2}{c^2 + 2cd + d^2}
\]

**Negative Signs** – If the negative sign is outside the parentheses, perform the operations inside the parentheses and carry out the negative sign to the final answer.

*Example:* \(-3^3 = -(3)(3)(3) = -(27) = -27\)

However, if the negative sign is inside the parentheses, the negative sign will be affected by the exponent.

*Example:* \((-3)^3 = (-3)(-3)(-3) = -27\)

If the negative sign is inside the parentheses and the exponent is an even number, the answer will be positive. If the exponent is an odd number, then the answer will be negative.

*Examples:*  
\(-2^3 = -8\leftarrow \text{Since the negative sign is outside the parentheses, carry it out to the final answer.}\)
\((-5)^2 = (-5)(-5) = 25\leftarrow \text{Since the negative sign is inside the parentheses, it needs to be carried out through the operation.}\)
\((-4)^2 = (-4)(-4) = 16\leftarrow \text{Even number of exponents, positive answer.}\)
\((-4)^3 = (-4)(-4)(-4) = -64\leftarrow \text{Odd number of exponents, negative answer.}\)
Negative Exponents – Whenever the problem or the answer to the problem contains negative exponents, they need to be changed to positive. An answer with negative exponents will most likely be counted wrong. To change negative exponents into positive exponents, get the reciprocal fraction. In simpler words, if the negative exponent is on the top, move it down; if the negative exponent is on the bottom, move it up.

*Examples:*

\[ x^{-2} \quad \leftarrow \text{Get the reciprocal, or move the negative exponent down.} \]

\[ x^{-2} = \frac{x^2}{1} = \frac{1}{x^2} \]

\[ 5y^{-3} \quad \leftarrow \text{Get the reciprocal of only the base with the negative exponent, the number stays in its place.} \]

\[ 5y^{-3} = \frac{5y^3}{1} = \frac{5}{y^3} \]

\[ \frac{a^2}{b^{-4}} \quad \leftarrow \text{Get the reciprocal of the base with the negative exponent, the base with the positive exponent stays in its place.} \]

\[ \frac{a^2}{b^{-4}} = \frac{a^2 \cdot b^4}{1} = a^2 \cdot b^4 \]
EXPONENTS – EXERCISES

Simplify:

1. \( x^5 \cdot x^7 \)
2. \( x^0 \) (for \( x \neq 0 \))
3. \( (a + b)^0 \) (for \( a + b \neq 0 \))

4. \( 5^{-3} \)
5. \( \frac{2^2}{2^3} \)
6. \( (3x)^2 \)

7. \( \left( \frac{5}{2} \right)^2 \)
8. \( \left( \frac{5}{2} \right)^2 \)
9. \( (x^3)^2 \)

10. \( \frac{x^2y}{xy} \)
11. \( (3^2)^2 \)
12. \( 9 \cdot 9^2 \)

13. \( -(a^2)^4 \)
14. \( (-a^2)^4 \)
15. \( \frac{c^7 \cdot c \cdot c^2}{c^5 \cdot c^2 \cdot c^3} \)

16. \( (a^2 b^2 c^2)^3 (-ab)^4 \)
17. \( (a^{-1})^{-1} \)
18. \( (4a^2 b)^2 (3a^2 b^3)^3 \)

19. \( \left( \frac{x^3}{x^2} \right)^3 \)
20. \( \left( \frac{1}{k} \right)^2 \)
21. \( (a^2 x^{-3})^2 \)

22. \( \frac{(a^2 x^{-3})^2}{(a^4 b^{-1})^3} \)
23. \( (-1)^3 \)
24. \( (-1)^9 \)

25. \( \left( \frac{8}{27} \right)^{-\frac{2}{3}} \)
EXPOENTS – ANSWERS TO EXERCISES

1. \( x^5 \cdot x^7 = x^{12} \)

2. \( x^0 = 1 \)

3. \((a + b)^0 = 1\)

4. \(5^{-3} = \frac{1}{5^3} = \frac{1}{125}\)

5. \(\frac{2^2}{2^3} = 2^{2-3} = 2^{-1} = \frac{1}{2}\)

6. \((3x)^2 = 3^2 \cdot x^2 = 9x^2\)

7. \(\left(\frac{5}{2}\right)^2 = \frac{2^2}{5^2} = \frac{4}{25}\)

8. \(\left(\frac{5}{2}\right)^2 = \frac{2^2}{5^2} = \frac{25}{4}\)

9. \((x^3)^2 = x^{(3)(2)} = x^6\)

10. \(\frac{x^2y}{xy} = x^{(2-1)} \cdot y^{(1-1)} = x^1 \cdot y^0 = x\)

11. \((3^2)^{-2} = 3^{-4} = \frac{1}{3^4} = \frac{1}{81}\)

12. \(9 \cdot 9^2 = 9^3\)

13. \(- (a^2)^4 = -(a^2)(a^2)(a^2)(a^2) = -a^8\)

14. \((-a^2)^4 = (-a^2)(-a^2)(-a^2)(-a^2) = a^8\)

15. \(\frac{c^7 \cdot c \cdot c^2}{c^5 \cdot c^2 \cdot c^3} = \frac{c^{10}}{c^{10}} = 1\)

16. \((a^2b^2c^2)(-ab)^4 = a^{10}b^{10}c^6\)

17. \((a^{-1})^{-1} = a\)

18. \((4a^2b^2)(3a^2b^3)^3 = 432a^{10}b^{11}\)

19. \(\left(\frac{x^3}{x^2}\right)^3 = \frac{x^9}{x^6} = x^3\)

20. \(\left(\frac{1}{k}\right)^{-2} = \frac{1^{-2}}{k^{-2}} = k^2\)

21. \((a^2x^{-3})^2 = a^{4}x^{-6} = \frac{a^4}{x^6}\)

22. \(\frac{(a^2x^{-3})^3}{(a^4b^{-3})^4} = a^{8}x^{-6} = \frac{b^3}{a^8x^6}\)

23. \((-1)^5 = -1\)

24. \((-1)^9 = -1\)

25. \(\left(\frac{8}{27}\right)^{\frac{2}{3}} = \left(\frac{3\sqrt[3]{8}}{3\sqrt[3]{27}}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{9}{4}\)