LINEAR FUNCTIONS

COMMON MISTAKES
Linear Functions - Definition, Notation and the Vertical Line Test

How to Understand the Definition and Notation

- **Linear Function** are relations in which exactly an x value is paired to exactly one y-value.

- Algebraically, these are labeled with alphabetic initials. The independent variable is inside the ( ).

- **Function Notation:**
  
  \[ f(x) = 3x - 7 \]

  where \( f \) = name of function and \( x \) = will be the independent variable.

Common Mistakes

- Not understanding the notation (not thinking of \( f(x) \) as \( y \))

- Being able to understand the notation so as to evaluate a function.

- Consider the problem...

  Find \( f(3) \) for \( f(x) = 2x + 9 \).

- **Incorrect:** \( f(3) \) **Does Not** mean
  
  \[ 3 = 2x + 9 \]

  -6 = 2x so \( f(3) = -3 \)

- **Correct:** \( f(3) = 2 \bullet 3 + 9 \) **Does** become \( f(3) = 15 \).

**Note:** Function Notation changes the ordered pair notation from (\( x, y \)) to (\( x, f(x) \)).
Linear Functions-Their Graph and Higher-Order Functions

How to Determine the Parent Functions

- **Parent Functions** are the ‘Basic’ functions we start with and then translate to form new functions.

- **The Degree of a function:**
  - Given the algebraic equation, the degree can be found by determining the greatest exponential power.
  - Graphically, we can test a function by drawing random vertical lines and intersecting the graph of the function (the line).
  - If the vertical line intersects in more than one point on the graph—the relation is **Not a function**.

- **Basic Parent Functions**
  - Linear: \( y = x \)
  - Quadratic: \( y = x^2 \)
  - Cubic: \( y = x^3 \)
  - Absolute Value: \( y = |x| \)

  And the list continues into higher order, more complex functions.

Common Mistakes

- Inability to recognize the Parent Function of complex functions.
- Graphically, not recognizing the shape common to the Basic Functions.
- Consider the problem: Identify the Parent-Function for this function…
  \[ y = 9x^2 - 3x + 4 \]

  - Incorrect: Parent function is NOT \( y=x \).

  - Correct: Parent Function is \( y = x^2 \).
Linear Functions-Domain and Range

How to correctly identify the Domain and Range.

- **Domain** - the possible x-values used in a function which give defined/acceptable y-values (INPUT)
- **Range** - can be found by determining what kind of answers are generated by functions. (OUTPUT)

Common Mistakes

- Confusing the Domain and the Range
- Given the function, identify the domain and range.

Correct:

D: \( \{x : -7 \leq x \leq 8\} \)
R: \( \{y : -9 \leq y < 3.5\} \)

Complete Manual: Linear Function Review.docx
To view; right click and open the hyperlink
Linear Functions- Add, Subtract, Multiply and Divide

How to Add, Subtract, Multiply, or Divide Functions

- Functions can be combined to create “New” functions.
  - If \( f(x) = x + 2 \) and \( h(x) = x^2 \)
  - Then \( h(x) = f(x) + g(x) \) is defined by
    \[
    (x + 2) + x^2 \quad \text{or} \quad x^2 + x + 2
    \]
  - Then \( h(x) = f(x) - g(x) \) is defined by
    \[
    x + 2 - x^2 \quad \text{or} \quad -x^2 + x + 2
    \]
  - Then \( h(x) = f(x) \cdot g(x) \) is defined by
    \[
    (x + 2) \cdot x^2 \quad \text{or} \quad x^3 + 2x^2
    \]
  - Then \( h(x) = \frac{f(x)}{g(x)}, g(x) \neq 0 \) is defined by
    \[
    \frac{x + 2}{x^2}
    \]

Common Mistakes

- Simplifying for the new function incorrectly by not distributing a negative sign or not multiplying correctly, etc.
  - If \( h(x) = f(x) - g(x) = 3x^2 - (-2x^2 + 7) \)
  - Incorrect: \( h(x) = x^2 - 7 \)
  - Correct: \( h(x) = 5x^2 - 7 \)
Linear Functions-Inverses

How to find the Inverse of a Linear Function

- Inverses “undo” another function.
- Graphically, a function’s inverse is the reflection of a function over the line y=x.
- Solving for the Inverse, \( f^{-1}(x) \):
  1. Rename ‘f(x) = ‘ as ‘y=‘.
  2. Switch the x and y in the equation.
  3. Solve for the y-variable.
  4. Replace the y with \( f^{-1}(x) \)

Common Mistakes

- Incorrectly solving for the inverse after the switching in Step 2.
- If \( f(x) = 9x - 2 \), what is \( f^{-1}(x) \)?
  - Incorrect: \( f^{-1}(x) = 2x - 9 \)
  - Correct: \( f^{-1}(x) = \frac{x + 2}{9} \)
  - Solution: 
    
    \[
    \begin{align*}
    y &= 9x - 2 \\
    x &= 9y - 2 \\
    x + 2 &= 9y
    \end{align*}
    \]

    Giving us \( f^{-1}(x) = \frac{x + 2}{9} \).
Linear Functions-Composite Functions

How to Solve Composite Functions

- Composite Functions combine functions in a special way to create a new function:

  **Notation:** \( h(x) = f(x) \circ g(x) \)

- To find \( h(x) \), \( h(x) = f(g(x)) \)
  1. Take the \( g(x) \) and put it in for \( x \) in the \( f(x) \)
  2. Solve for \( h(x) \)

Ex. \( f(x) = \sqrt{x} \) And \( g(x) = (x+2)^2 \)

\[ h(x) = \sqrt{(x+2)^2} = x + 2 \]

Common Mistakes

- Incorrectly substituting into the wrong function.
- Algebraic errors in simplifying.
- Consider: \( f(x) = \sqrt{x} \) and \( g(x) = 2x^2 - 1 \)

Find…………. \( h(x) = (f \circ g)(x) \)

  **Incorrect:** \( h(x) = \sqrt{2x^2 - 1} \)

  **Correct:** \( h(x) = f(2x^2 - 1) \)

  **Solution:**
  Substitute……… \( h(x) = \sqrt{2x^2 - 1} \)
  Sub Again……… \( h(x) = \sqrt{2x^2 - 1} \)